Rational Functions(1)

Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

1. Determine the domain of definition D of the function f and calculate the limits of f at the boundaries of D.

$x^2 - 1 \neq 0 \quad ; \quad x^2 \neq 1 \quad ; \quad x \neq \pm 1$

So $D =]-\infty; -1[\cup]-1; 1[\cup]1; +\infty[$

$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2}{x^2} = 1$ so $y = 1$ is a horizontal asymptote.

$\lim_{x \to 1^+} f(x) = \frac{1}{0^+} = +\infty \quad ; \quad \lim_{x \to 1^-} f(x) = \frac{1}{0^-} = -\infty$

So $x = 1$ is a vertical asymptote.

$\lim_{x \to -1^+} f(x) = \frac{1}{0^-} = -\infty \quad ; \quad \lim_{x \to -1^-} f(x) = \frac{1}{0^+} = +\infty$

So $x = -1$ is a vertical asymptote

Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

2. Study the parity of f and interpret graphically.

Domain is $D =]-\infty;-1[\cup]-1;1[\cup]1;+\infty[$ is centered at O.

$$f(-x) = \frac{(-x)^2}{(-x)^2-1} = \frac{x^2}{x^2-1} = f(x)$$

So f is an even function.

Graphical interpretation:

$(y'y)$ is an axis of symmetry

Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

3. Study the sign of $f(x)$. Interpret graphically.

x		-1		0		1	
x^2	$+$	‖	$+$	0	$+$	‖	$+$
x^2-1	$+$	‖	$-$		$-$	‖	$+$
$f(x)$	$+$	‖	$-$	0	$-$	‖	$+$

$x \in]-\infty;-1[\cup]1;+\infty[: f(x) > 0$

(C) is above $(x'x)$

$x \in]-1;0[\cup]0;1[: f(x) < 0$

(C) is below $(x'x)$

$x = 0 : f(x) = 0$

(C) cuts $(x'x)$ at (0;0)

Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

4. Calculate $f'(x)$ and set up the table of variations of the function f.

$$f'(x) = \frac{2x(x^2-1)-x^2\times 2x}{(x^2-1)^2} = \frac{2x(x^2-1-x^2)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$$f'(x) = 0 \ ; \ -2x = 0 \ ; \ x = 0 \ ; \ y = 0$$

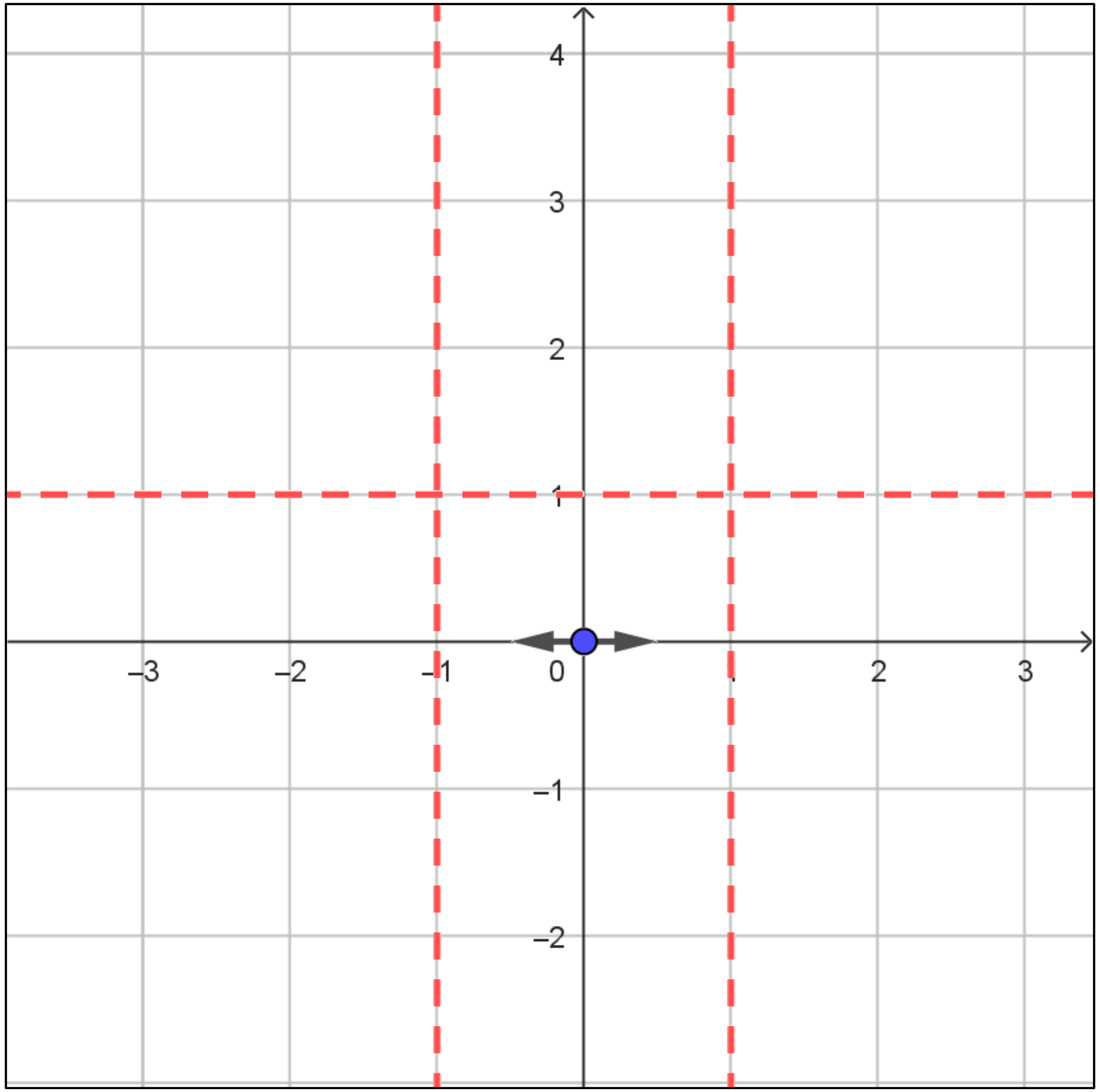
x		-1		0		1	
$f'(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$

Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

5. Plot (C).

- Plot the asymptotes.
- Plot the extrema
- P.P. : (0;0)
- Draw based on the table of variations.

x		-1		0		1	
$f'(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$



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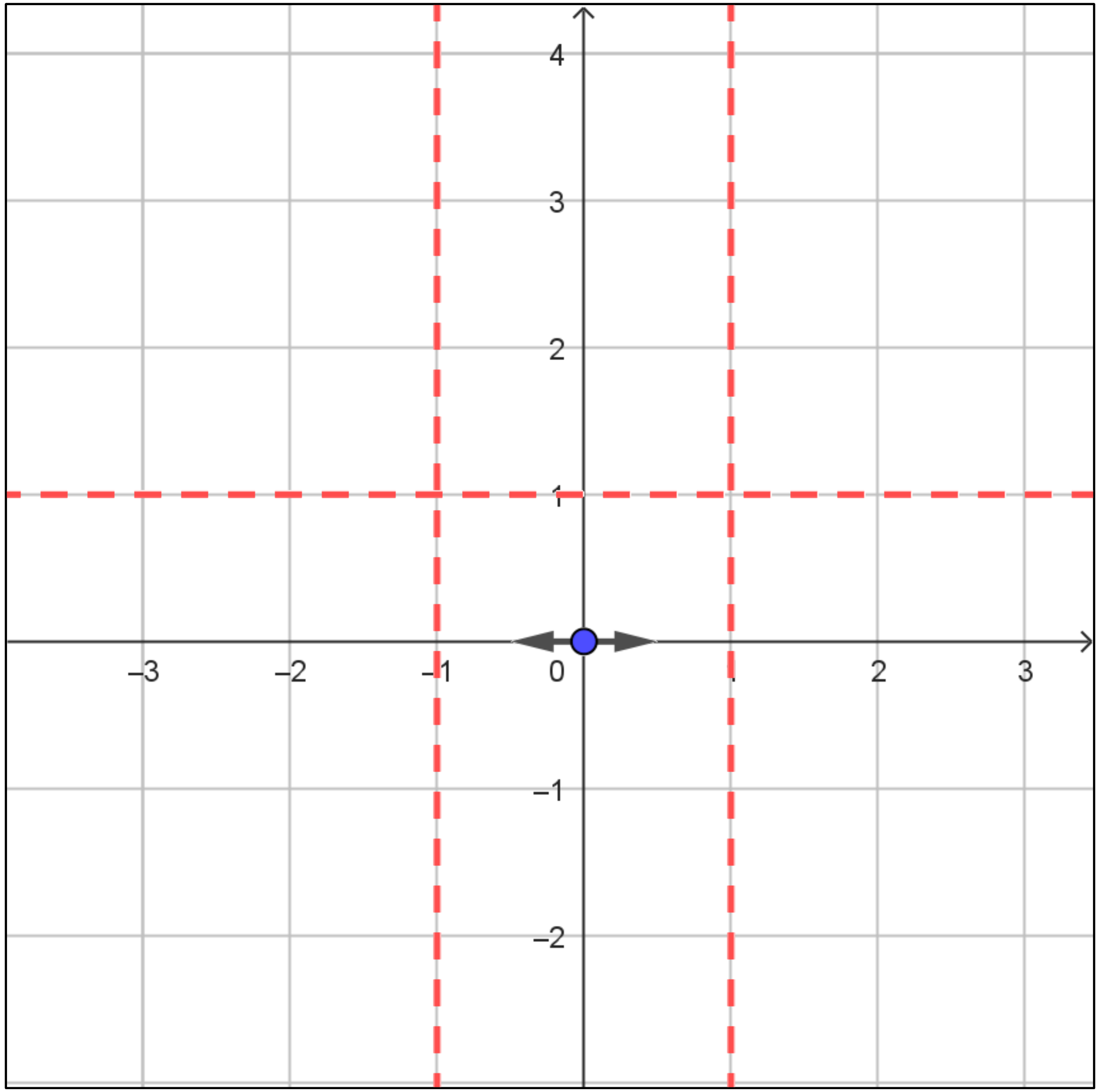
5. Plot (C).

x		-1		0		1	
$f'(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$

The function starts from the horizontal asymptote then moves far from it.



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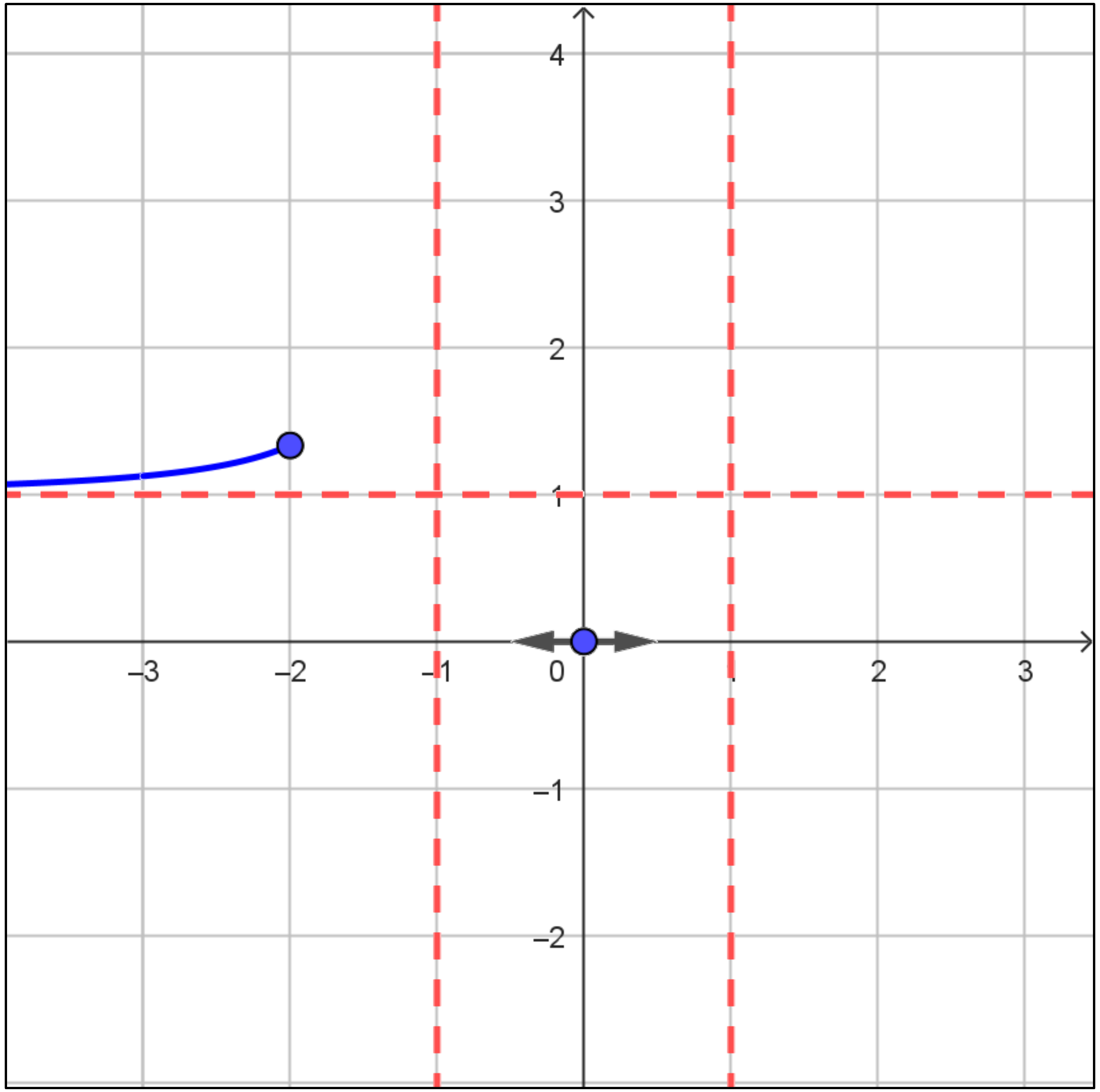
an orthonormal system $(O; \vec{i}; \vec{j})$.

5. Plot (C).

To work precisely, we can plot a helping point:

For $x = -2$; $y = \frac{4}{4-1} = \frac{4}{3} \approx 1.3$



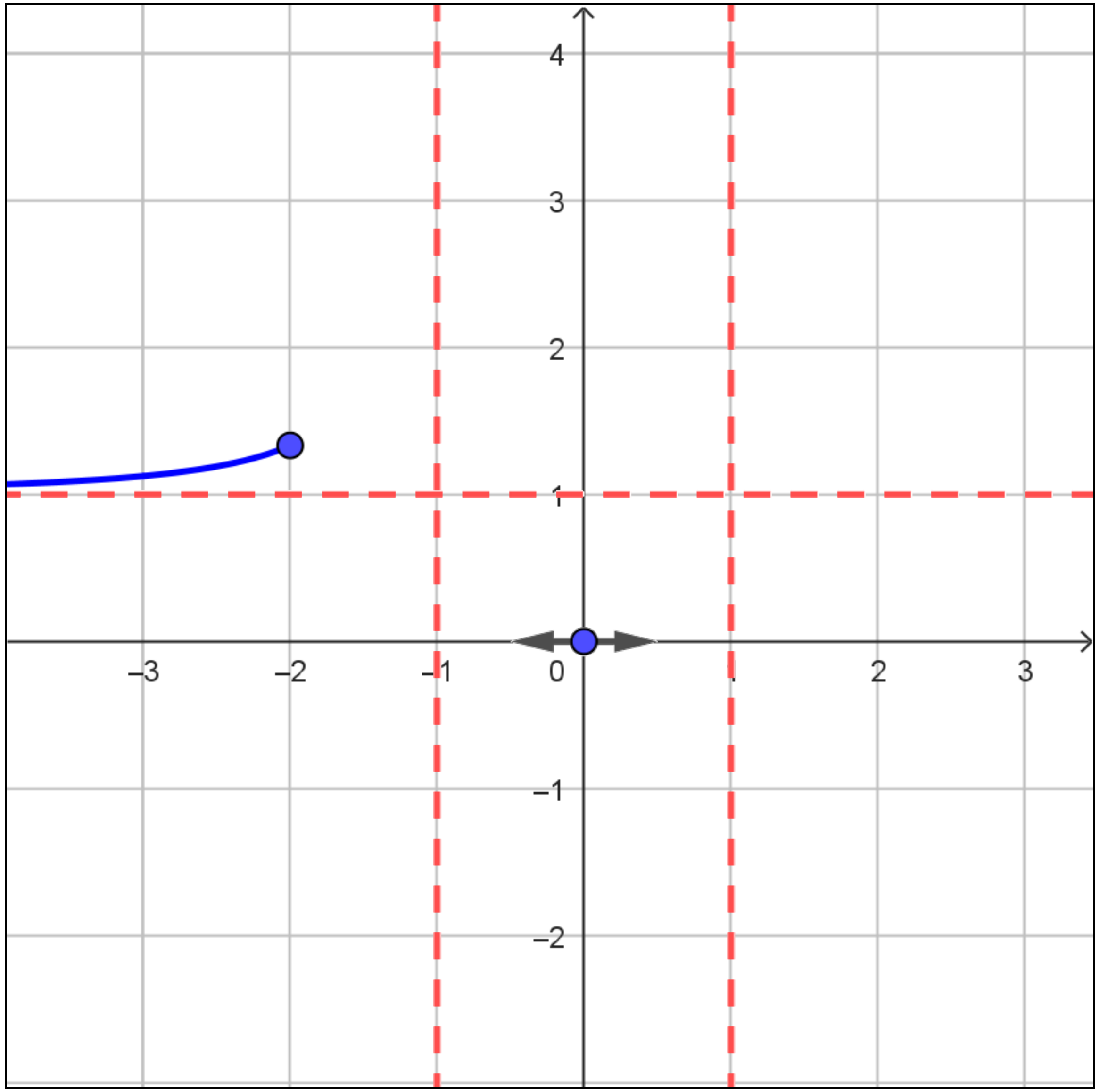
The function starts from the horizontal asymptote then moves far from it.

Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

5. Plot (C).

x		-1		0		1	
$f'(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$

The function moves toward the vertical asymptote $x = -1$.



Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

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x		-1		0		1	
$f'(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$

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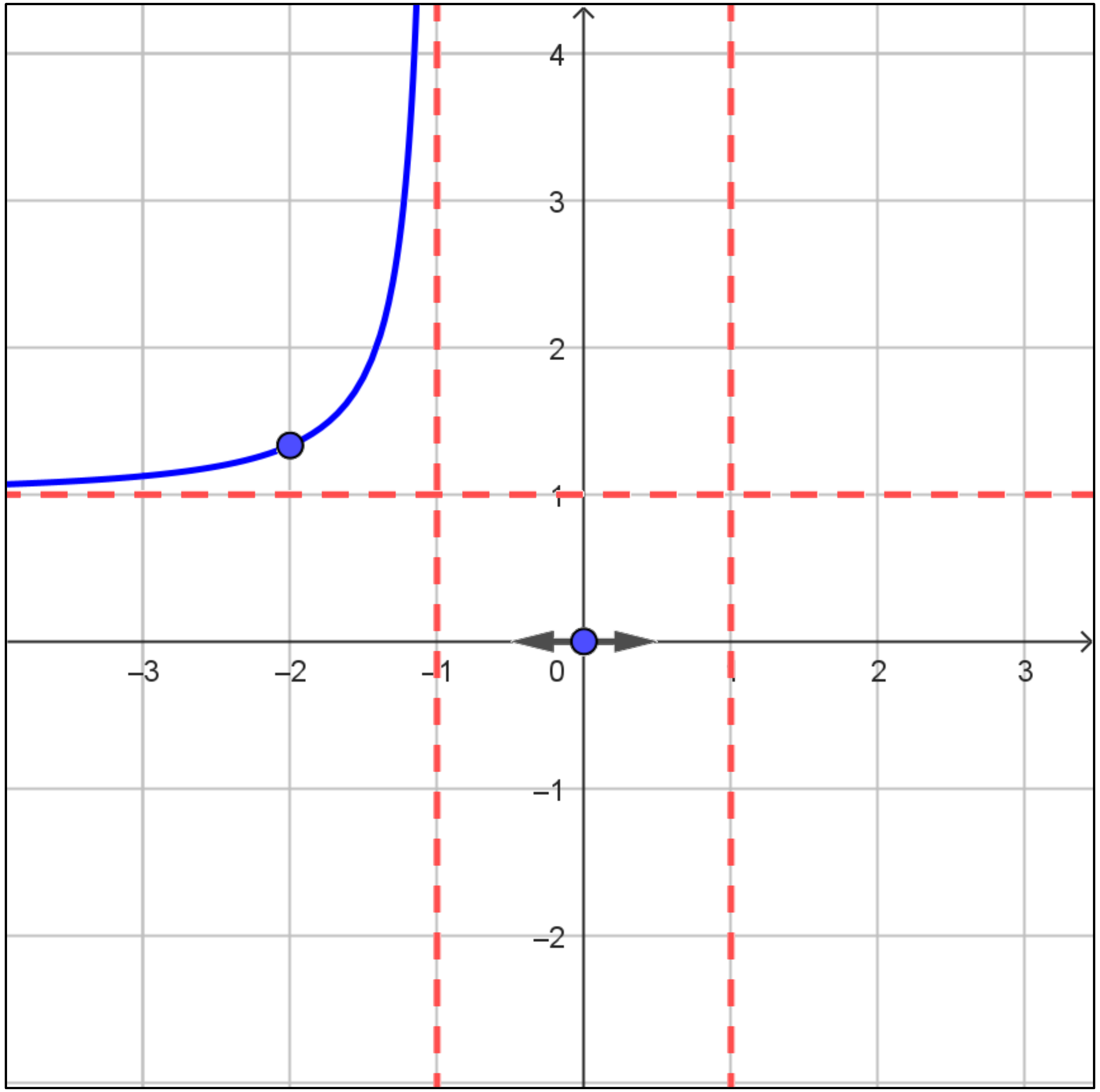

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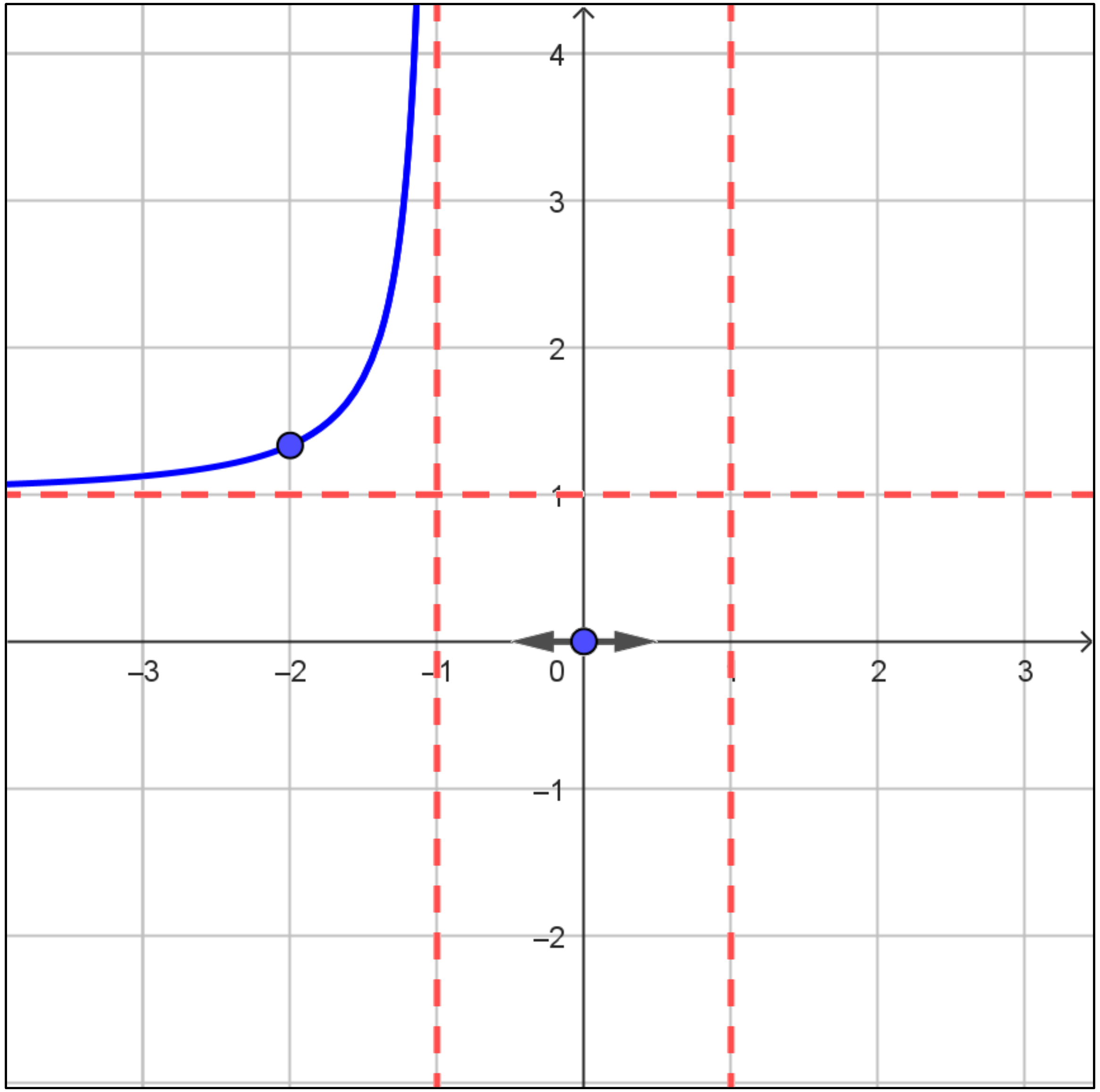
x		-1		0		1	
$f'(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$

The function moves away the vertical asymptote $x = -1$ toward the extremum (0,0)



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5. Plot (C).

x		-1		0		1	
$f'(x)$	$+$	||	$+$	0	$-$	||	$-$
$f(x)$	$1 \nearrow +\infty$	||	$-\infty \nearrow$	0	$\searrow -\infty$	||	$+\infty \searrow 1$

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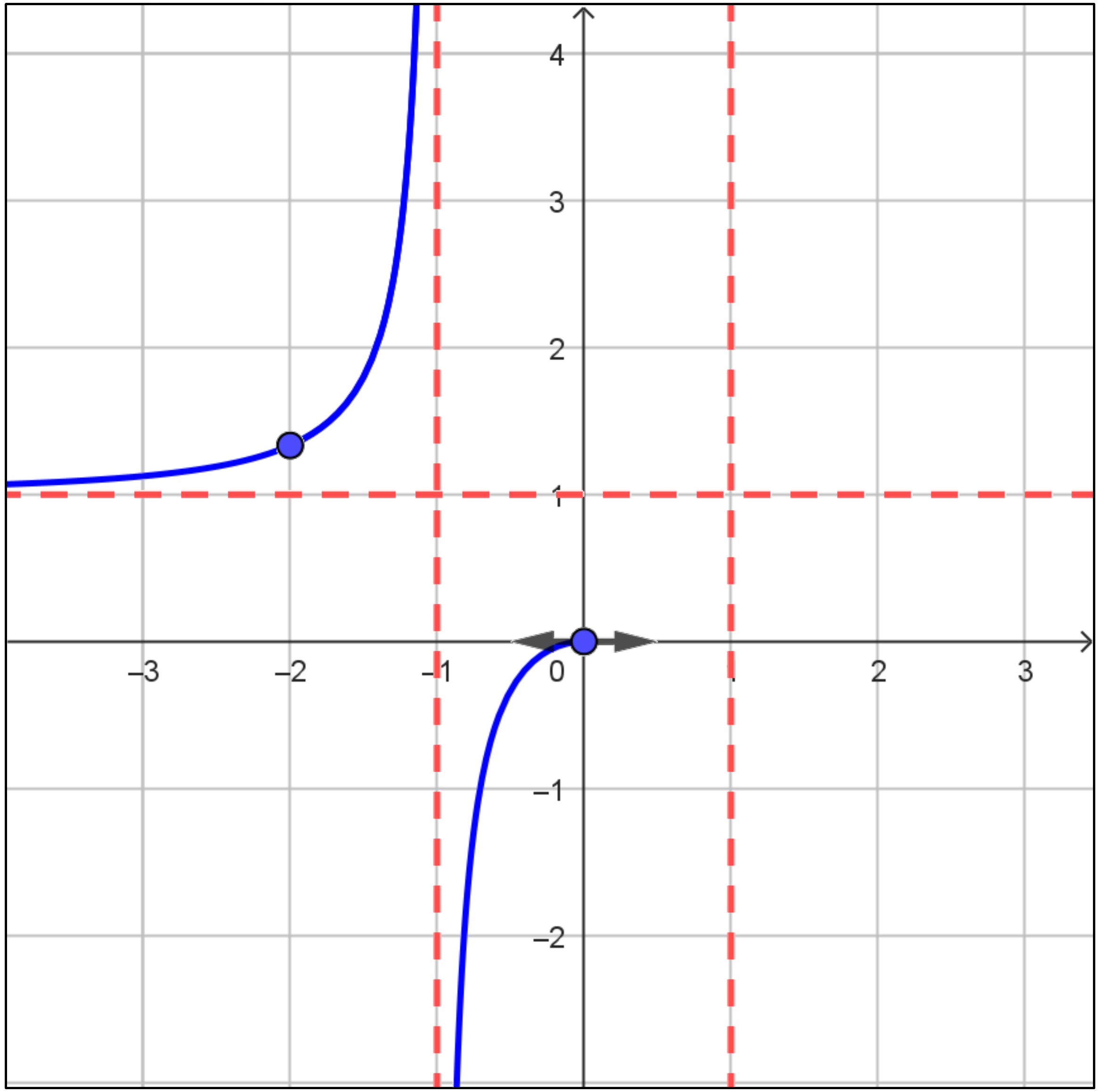

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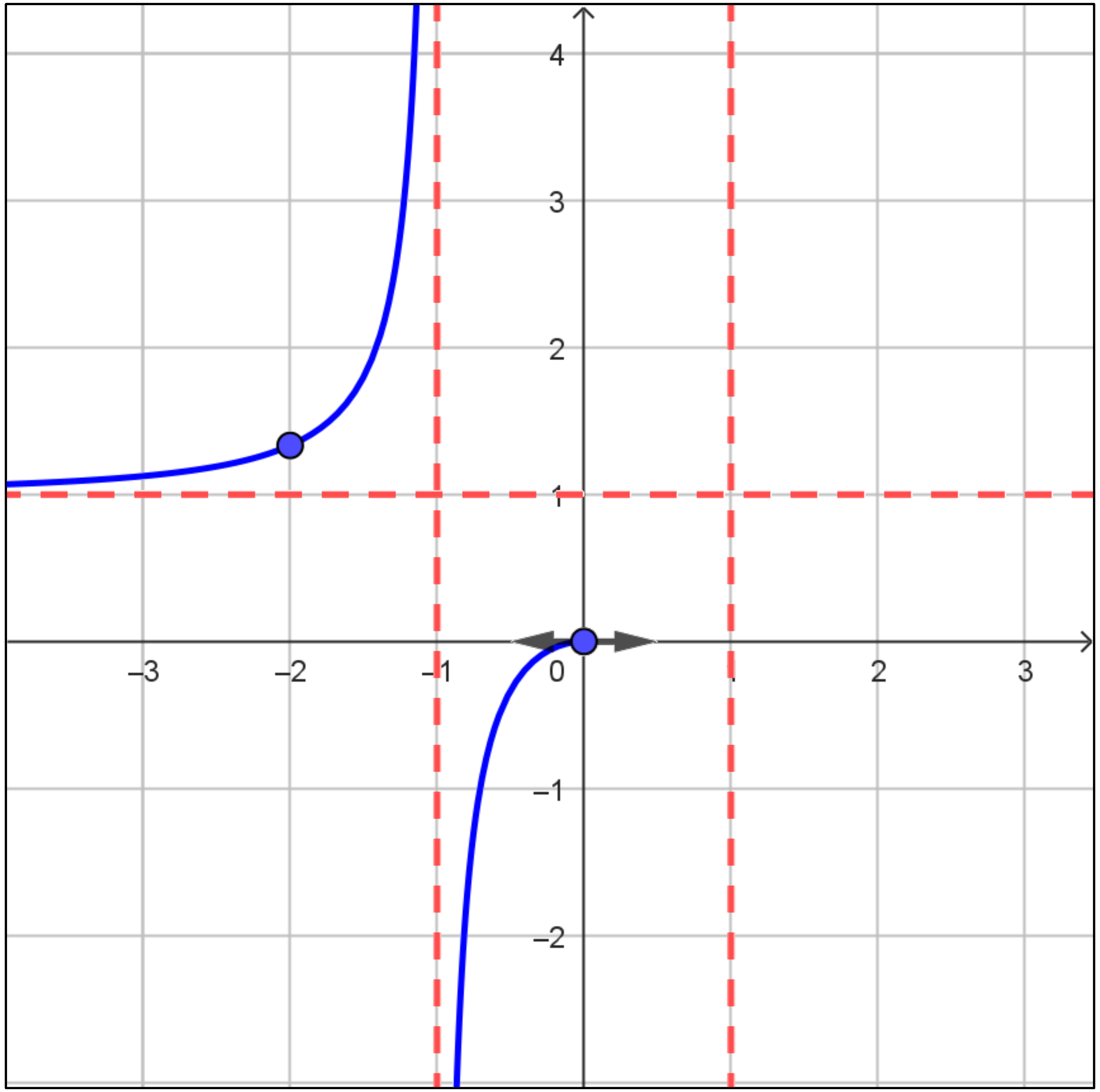
x		-1		0		1	
$f'(x)$	$+$	‖	$+$	0	$-$	‖	$-$
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$



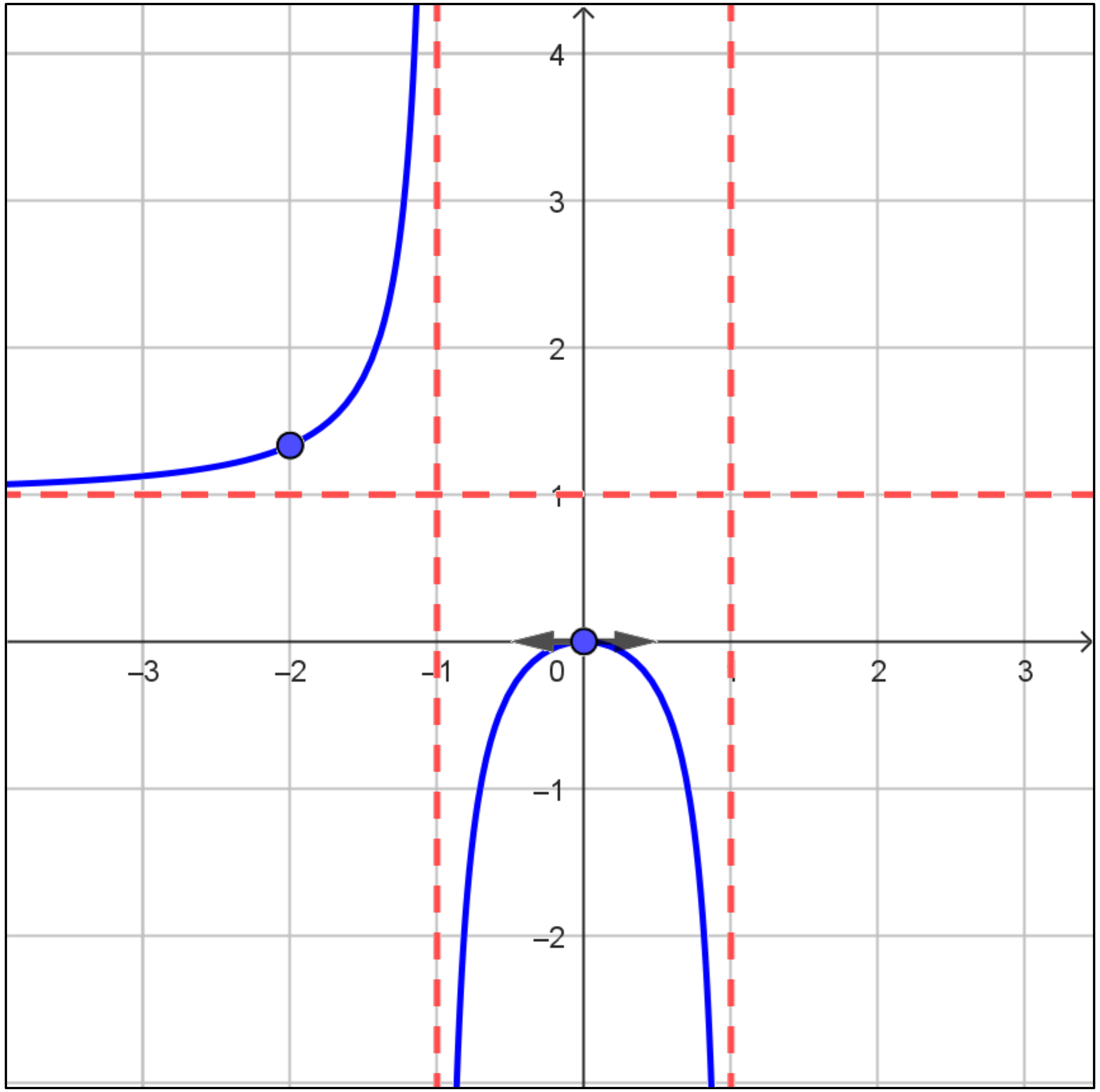
The function moves toward the vertical asymptote $x = 1$ from the extremum (0,0)

Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative cu[rve in] an orthonormal system $(O;\vec{i};\vec{j})$.

5. Plot (C).

x		-1		0		1	
$f'(x)$	+	‖	+	0	−	‖	−
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$

The function moves toward the vertical asymptote $x = 1$ from the extremum (0,0)



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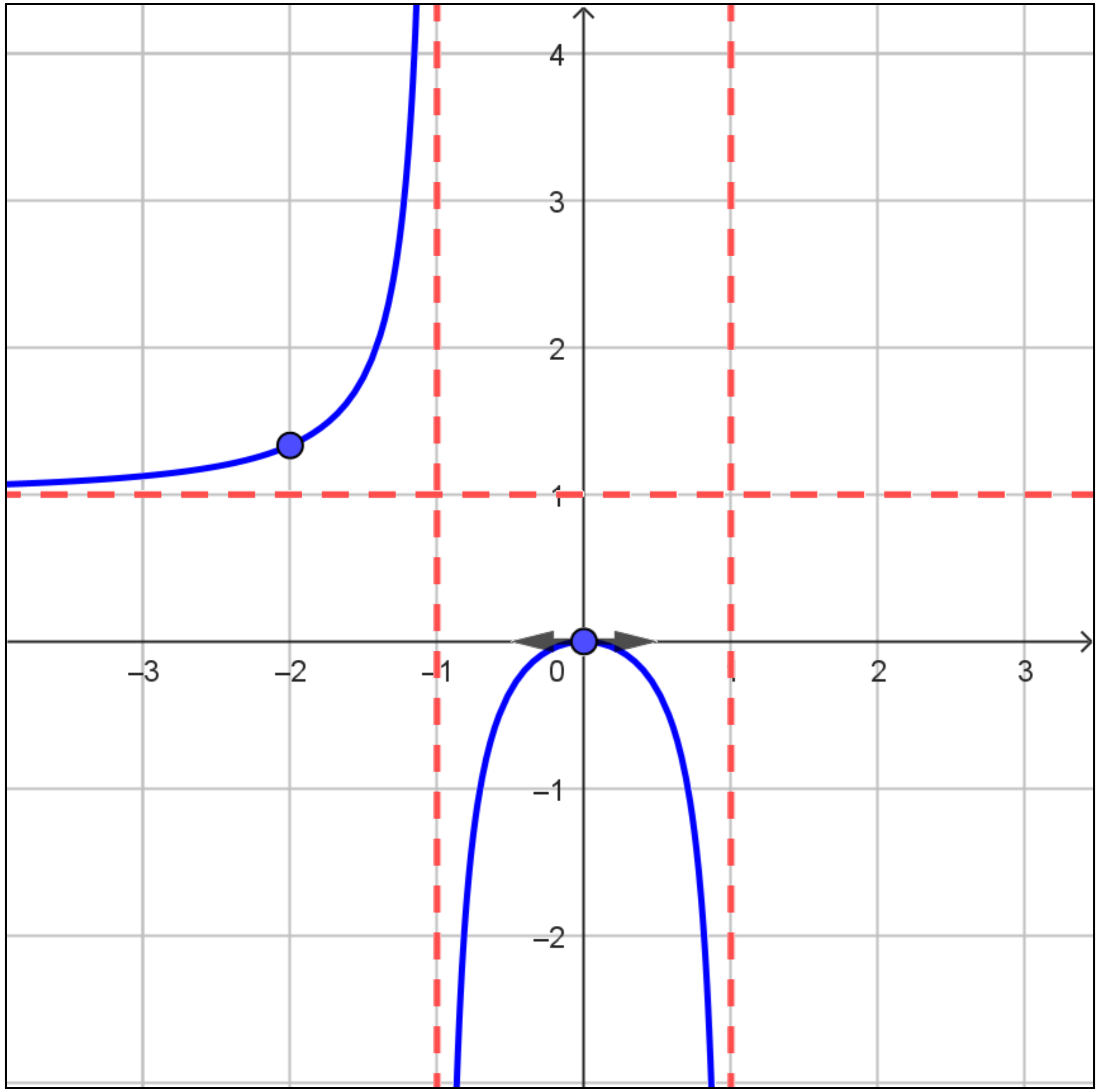
$f(x)$	$1 \nearrow +\infty$	‖	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow 1$

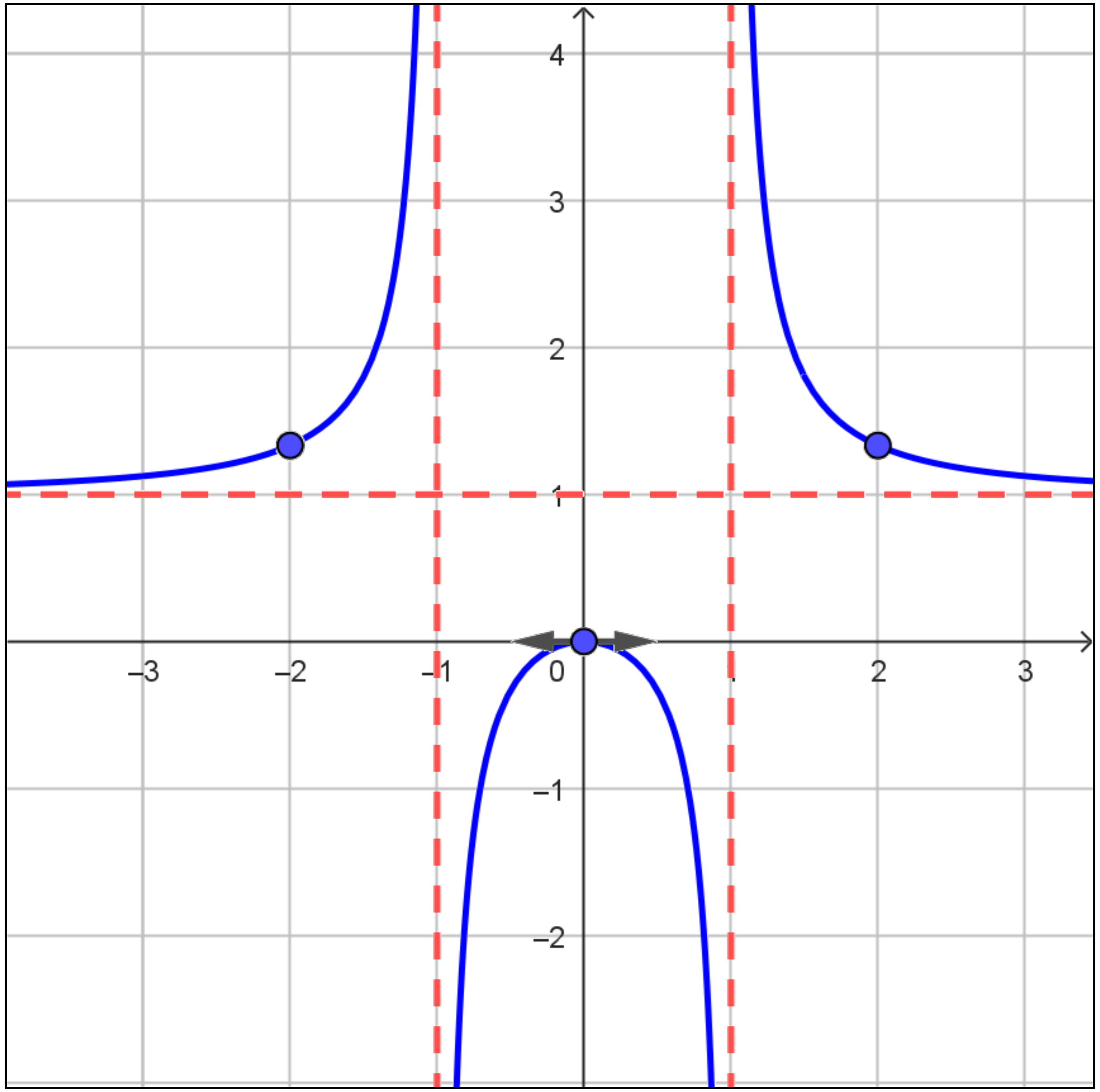
The function moves away the vertical asymptote $x = 1$ toward the horizontal asymptote $y = 1$



Consider the function f defined by $f(x) = \frac{x^2}{x^2-1}$. (C) its representative cu... an orthonormal system $(O; \vec{i}; \vec{j})$.

5. Plot (C).

Place a helping point to draw precisely.
You can benefit from (y'y) since it is an axis of symmetry



The function moves away the vertical asymptote x = 1 toward the horizontal asymptote y = 1